

Squeeze processes in narrow circular damper with highly compressible porous layer imbibed with liquids

Maria-Brindusa Ilie^a, Mircea D. Pascovici^a and Victor G. Marian^a

^a Department of Machine Elements and Tribology, University POLITEHNICA of Bucharest,
Splaiul Independentei, 313, 060042, Bucharest, ROMANIA

milie@omtr.pub.ro

Abstract

Squeeze film dampers are widely used technical components for turbomachinery as a means to cut down the amplitude of rotor vibration owing to unbalance [1]. This paper proposes a different type of circular dampers using highly compressible porous layers (HCPL) imbibed with Newtonian liquids. The elastic forces of the HCPL solid phase are negligible compared to the hydrodynamic forces generated within the porous layer [2]. Such processes were named ex-poro-hydrodynamic (XPHD). The Kozeny-Carman equation was used to compute permeability in function of porosity / compacticity. Analytic and numeric solutions were performed for the impact process of the partial and full narrow circular dampers in XPHD conditions. The results were compared with the case of the classical squeeze film damper. The damping capacity of a HCPL imbibed with Newtonian liquid was found to be considerably greater than that of the Newtonian liquid layer.

Keywords: circular damper; porous medium; compliant layer; damping capacity;

1. INTRODUCTION

The damping under impact conditions in ex-poro-hydrodynamic regime of the highly compressible porous layers imbibed with Newtonian liquids was previously studied theoretically and experimentally for the aligned plates geometries [2] and sphere to plan contact [3]. The circular dampers are of particular interest due to the similarity with the commonly used squeeze film dampers [1]. The squeeze film dampers main advantages consist on the capability of reducing the effects of unbalance, i.e. the amplitude of rotor vibrations at resonance and the level of forces transmitted at supports under normal operating conditions, and in preventing the occurrence of non-synchronous instabilities or limiting their effects [1].

The present paper approaches for the first time the behaviour of an XPHD narrow circular damper subjected to impact. Analytic and numeric solutions were developed for the impact process of the partial and full narrow circular dampers in XPHD conditions. The results are compared with the case of narrow classical squeeze film damper. Attractive results can lead in designing more efficient XPHD squeeze dampers. Imagining such dampers as self-contained and using water as imbibing liquid make these devices eco-friendly.

2. THEORY

2.1 Main assumptions and problem formulation

The geometry of a circular squeeze damper is represented in Fig. 1. Assumptions applied in previous papers [2,3,4] will be considered for the present study:

1. The liquid is Newtonian and the flow in the porous layer is laminar and isothermal/isoviscous.
2. The elastic forces of HCPL are negligible compared to the hydrodynamic forces generated within the porous layer.
3. The pressure across the HCPL thickness is constant.

4. The local deformation is present only on the normal direction of the porous layer.

The solid mass is conserved throughout the process of layer deformation. For the sake of clarity the term of *local compacticity*, σ , is introduced, i.e. the instantaneous solid fraction. Consequently, the compacticity is related to porosity, ε , $\sigma = 1 - \varepsilon$. This assumption was accepted in all the papers on XPHD lubrication:

$$\sigma h = \sigma_i h_i = \sigma_0 h_0 \quad (1)$$

where σ_0 and h_0 are the uncompressed compacticity (solid fraction) and layer thickness.

5. The HCPL permeability variation is correlated with compacticity according to Kozeny-Carman law:

$$\phi = \frac{D(1-\sigma)^3}{\sigma^2} \quad (2)$$

Specific assumptions of the analyzed problem:

6. The HCPL dimensions, $B/d \leq 0.7$, lead to the concept of narrow bush for which the axial flow, in z direction, is prevailing.
7. The flow analysis is performed for both the compressed area, $\Omega = 180^\circ$, and the entire circumference of the bush, $\Omega = 360^\circ$.
8. In the case of $\Omega = 360^\circ$ it is acknowledged that depressions appear in the divergent area and that the cavitation effects are not be taken into consideration.
9. Issues related to the existence of a sealing system are not considered.
10. The HCPL is fitted between the journal and the bush considering a pre-tightening in thickness h_i and a characteristic compacticity, σ_i .

Because the porous layer is considered thin ($h_i \ll d$) and the journal and housing are circular and rigid, the thickness variation of HCPL is:

$$h = h_i - e \cos \theta \quad (3)$$

In dimensionless form relation (3) becomes:

$$H = 1 - \bar{e} \cos \theta \quad (3')$$

Also, the minimal HCPL thickness is:

$$H_m = 1 - \bar{e} \quad (3'')$$

From the flow conservation condition on z direction, to a certain section, θ , of the HCPL results:

$$\varepsilon V \cos \theta z = -\frac{\phi h}{\eta} \frac{dp}{dz} \quad (4)$$

Considering Eqs. (1), (2) and (3) in Eq. (4) and simplifying, results the pressure differential equation:

$$\frac{dp}{dz} = -\frac{\eta V \sigma_i^2 \cos \theta}{D h_i (1 - \sigma_i - \bar{e} \cos \theta)^2 (1 - \bar{e} \cos \theta)} z \quad (5)$$

Integrating Eq. (5) and applying the boundary condition $p = p_0$ at $z = B/2$, one can obtain the pressure distribution:

$$p = p_0 + \frac{\eta V \sigma_i^2}{2 D h_i (1 - \sigma_i)^2} \left(\frac{B^2}{4} - z^2 \right) f(\theta) \quad (6)$$

where

$$f(\theta) = \frac{\cos \theta}{\left(1 - \frac{\bar{e}}{1 - \sigma_i} \cos \theta \right)^2 (1 - \bar{e} \cos \theta)} \quad (7)$$

The resultant force for squeeze at constant velocity is calculated:

$$F = 4 \int_0^{B/2} \int_0^{\Omega/2} p \frac{d}{2} \cos \theta dz d\theta \quad (8)$$

Performing the first integration and considering $p_0 = 0$, the contact force for squeeze at constant velocity becomes:

$$F = \frac{\eta V \sigma_i^2 B^3 d}{12 D h_i (1 - \sigma_i)^2} I \quad (9)$$

where

$$I = \int_0^{\Omega/2} \cos \theta f(\theta) d\theta \quad (10)$$

2.2 Analytic solutions

Unfortunately one cannot obtain a closed-form solution for Eq. (9) in order to get further the impact force and its variation. Therefore, approximate solutions were developed in two distinct cases:

A. The case of $\sigma_i \ll 1$

It has been noticed in recent applications [3] that the XPHD impact damping capacity of HCPL materials require small initial compacticities, $\sigma_i = 0.05 \div 0.1$. Considering this observation, $\frac{\bar{e}}{1 - \sigma_i} \cong \bar{e}$, Eq. (7) becomes:

$$f_a(\theta) = \frac{\cos \theta}{(1 - \bar{e} \cos \theta)^3} \quad (11)$$

Due to this approximation the model is restricted to relatively small eccentricities, i.e. $\bar{e} < 0.5$. Considering these assumptions and performing the integration of the pressure distribution for the compressed area of HCPL, $\Omega = 180^\circ$, one finally obtains the force at constant velocity:

$$F = \frac{\eta V \sigma_i^2 B^3 d}{24 D h_i (1 - \sigma_i)^2} \left[\frac{3\bar{e}}{(1 - \bar{e}^2)^2} + \frac{2\bar{e}^2 + 1}{(1 - \bar{e}^2)^{5/2}} \cos^{-1}(-\bar{e}) \right] \quad (12)$$

To facilitate the calculation of impact force the Booker approximation for the journal bearings hydrodynamic model [5] is used. Thus, the force at constant velocity for the compressed area of HCPL is:

$$F = \frac{\pi\eta V\sigma_i^2 B^3 d}{48Dh_i(1-\sigma_i)^2(1-\bar{e})^{5/2}} \quad (13)$$

Using the Bowden and Tabor model for hydrodynamic squeeze under impact [6] and considering $V = -\frac{dH_m}{dt}h_i = \frac{d\bar{e}}{dt}h_i$ the velocity variation will be computed from the following equation:

$$MdV = -Fdt \quad (14)$$

Hence, using Eq. (14) and the relation of force given by Eq. (13) the velocity variation is obtained as function of the minimum layer thickness, H_m :

$$V = V_0 + \frac{\pi\eta\sigma_i^2 B^3 d}{48MD(1-\sigma_i)^2} \int_1^{H_m} \frac{dH_m}{H_m^{5/2}} \quad (15)$$

Finally, performing the integral, the velocity variation during impact process, in dimensionless form, is:

$$\bar{V} = 1 + \frac{\pi Po \sigma_i^2}{72\bar{M}(1-\sigma_i)^2} \left(1 - \frac{1}{H_m^{3/2}} \right) \quad (16)$$

The squeeze under impact process is entirely performed in XPHD conditions if at the end of the approaching process when velocity becomes zero the dimensionless minimum thickness, H_m , is greater than the imposed initial compacticity, σ_i ($H_m > \sigma_i$, see Eq. (1)). Also, the maximum allowable dimensionless impulse, \bar{M}_{\max} , results for the squeeze process, performed entirely in XPHD conditions, applying the boundary condition $\bar{V} = 0$ when $H_m = \sigma_i$ in Eq. (16):

$$\bar{M}_{\max} = \frac{\pi Po \sigma_i^2}{72(1-\sigma_i)^2} \left(\frac{1}{\sigma_i^{3/2}} - 1 \right) \quad (17)$$

Introducing in Eq. (13) the velocity variation during impact process given by Eq. (16), one obtains the impact force variation, in dimensionless form, as function of H_m :

$$\bar{F}_s = \frac{\pi \text{Po} \sigma_i^2 (B/d)^2}{48(1-\sigma_i)^2 H_m^{5/2}} \left[1 + \frac{\pi \text{Po} \sigma_i^2}{72\bar{M}(1-\sigma_i)^2} \left(1 - \frac{1}{H_m^{3/2}} \right) \right] \quad (18)$$

Following the same procedure, one can find the maximum allowable dimensionless impulse, \bar{M}_{\max} , for a full, $\Omega = 360^\circ$, circular damper in a squeeze XPHD process:

$$\bar{M}_{\max} = \frac{\pi \text{Po} \sigma_i^2}{24(1-\sigma_i)^2} \frac{1-H_m}{(2H_m - H_m^2)^{3/2}} \quad (19)$$

Also the dimensionless impact force for the entire circumference results:

$$\bar{F}_s = \frac{\pi \text{Po} \sigma_i^2 (B/d)^2 [1 + 2(1-H_m)^2]}{24(1-\sigma_i)^2 (2H_m - H_m^2)^{5/2}} \left[1 - \frac{\pi \text{Po} \sigma_i^2}{24\bar{M}(1-\sigma_i)^2} \frac{1-H_m}{(2H_m - H_m^2)^{3/2}} \right] \quad (20)$$

B. The case of $\bar{e} \ll 1$

This assumption is suitable for high frequency operating conditions. When the damper is working with high frequency the eccentricity reduction is normal. One can estimate that the eccentricities lie in the range of $\bar{e} \cong 0.05 \div 0.1$. In this case Eq. (7) becomes:

$$f_b(\theta) = \frac{(1 + \bar{e} \cos \theta) \cos \theta}{\left(1 - \frac{\bar{e}}{1-\sigma_i} \cos \theta \right)^2} \quad (21)$$

If noting $E = \frac{\bar{e}}{1-\sigma_i}$ then Eq. (21) can be rewritten as:

$$f_b(\theta) = \frac{\cos \theta + \bar{e} \cos^2 \theta}{(1 - E \cos \theta)^2} \quad (21')$$

Considering this case of small eccentricities, one integrating the pressure distribution over the compressed area of HCPL (i.e. $\Omega = 180^\circ$) using Eq. (21'), the integral I given by Eq. (10) becomes:

$$I_b = \int_0^{\pi/2} \cos \theta f_b(\theta) d\theta \quad (22)$$

The integral I_b is solved using Booker integrals method [7], and one obtains:

$$I_b = \frac{\cos^{-1}(-E)}{E^2(1-E^2)^{3/2}} \left[(2E^2 - 1) + e \left(3E - \frac{2}{E} \right) \right] + \frac{\pi(1-E^2) + 2E}{2E^2(1-E^2)} - e \frac{E^3 + \pi E^2 - 2E - \pi}{E^3(1-E^2)} \quad (23)$$

Finally, the force at constant velocity over the compressed area of HCPL is:

$$F = \frac{\eta V \sigma_i^2 B^3 d}{12 D h_i (1 - \sigma_i)^2} I_b \quad (24)$$

Using Eq. (24) and the Bowden and Tabor model for squeeze under impact [6] given by Eq. (14), the velocity variation during impact process as function of E results:

$$V = V_0 - \frac{\eta \sigma_i^2 B^3 d}{12 M D (1 - \sigma_i)} \int_0^E I_b dE \quad (25)$$

The velocity variation in dimensionless form is:

$$\bar{V} = 1 - \frac{\text{Po} \sigma_i^2}{12 \bar{M} (1 - \sigma_i)} \int_0^E I_b dE \quad (26)$$

The maximum allowable dimensionless impulse, \bar{M}_{\max} , results for the squeeze process applying the boundary condition $\bar{V} = 0$ when $H_m = \sigma_i$ in Eq. (26):

$$\bar{M}_{\max} = \frac{\text{Po} \sigma_i^2}{12 (1 - \sigma_i)} \int_0^1 I_b dE \quad (27)$$

Introducing the velocity variation expressed by Eq. (26) in Eq. (24), the impact force in dimensionless form results:

$$\bar{F}_s = \frac{\text{Po} \sigma_i^2 (B/d)^2 I}{12 (1 - \sigma_i)^2} \left(1 - \frac{\text{Po} \sigma_i^2}{12 \bar{M} (1 - \sigma_i)} \int_0^E I_b dE \right) \quad (28)$$

2.3 Numeric solutions

In order to model the impact squeeze process for a narrow circular damper for the range of all eccentricities and initial compacticities, the integral I given by Eq. (10) is computed numerically using the Romberg integration method. Following the same procedure as for the

analytic case using Bowden and Tabor model for squeeze under impact, the variations of the velocity and impact force are computed.

The results mach perfectly with the analytical solution in the case of $\bar{e} \ll 1$ and are in good agreement for $\sigma_i \ll 1$ (a difference arises due to the approximation of Eq. (7) given by Eq. (11)). The computation was made for both partial, $\Omega = 180^\circ$, and full, $\Omega = 360^\circ$, squeeze damper.

3. RESULTS AND DISCUSSIONS

The results are presented in terms of dimensionless impact force for both analytical cases, $\sigma_i \ll 1$ and $\bar{e} \ll 1$, and for the numerical solution. Fig. 2 shows the impact force variation as function of minimum dimensionless HCPL thickness, H_m , for different dimensionless impulses \bar{M} for both partial and full circular damper. One can see that the dimensionless impact force \bar{F}_s increases with the dimensionless impulse \bar{M} at a given small compacticity $\sigma_i = 0.08$. The computation is achieved using the analytic approach of small initial compacticities, $\sigma_i \ll 1$.

In the other case of small eccentricities one can observe the same increase of the dimensionless impact force \bar{F}_s but for considerably greater impulses \bar{M} . Fig. 3 shows the variation of the dimensionless impact force for different dimensionless impulses at a given initial compacticity $\sigma_i = 0.8$. The results are obtained using the analytic approach of small eccentricities, $\bar{e} \ll 1$. It is important to note that for small eccentricities correspond very high initial compacticities.

Adapting for XPHD conditions the Knox model [8] used in the analysis of the hydrodynamic squeeze process in magnetic bearings one can observe an acceptable match of the results for the cases of small eccentricities and small initial compacticities.

The numerical solution that disregards any simplifying assumption covers the entire range of values for initial compacticities and eccentricities. Further, the variation of dimensionless impact force as function of minimum HCPL thickness, H_m , for a given impulse $\bar{M} = 20000$ and different initial compacticities is presented in Fig. 4. One can remark that the optimal value for compacticity can be found around $\sigma_i = 0.5$. The results obtained for the dimensionless impact force are generally smaller for $\Omega = 360^\circ$ compared to $\Omega = 180^\circ$ wrapping angle. The maximum value of the impact force decreases from very small eccentricities till the optimal value of compacticity and then increases as the initial compacticities increase.

A comparison between the analytic and numeric results of the dimensionless impact force as function of minimum HCPL thickness, H_m , for a given impulse $\bar{M} = 500$ and different small initial compacticities is presented in Fig. 5. One can observe a good correlation between the two approaches.

The previous papers regarding XPHD processes [2, 4, 9-11] have pointed out the drastic increase of load carrying capacity and, also, the significant damping capacity of porous layers in XPHD process, compared to hydrodynamic (HD) regime. The XPHD regime gives a load capacity of 2-3 orders of magnitude greater than the load capacity of HD regime. Hence, it is interesting to compare the performance of the squeeze process of a circular damper under impact in XPHD regime with the HD regime. Therefore, using the model of squeeze under impact in HD regime presented in Appendix, the dimensionless impact force for both 180° and 360° arc bush from Eqs. (A4) and (A9) are:

$$\bar{F}_s^{HD} = \frac{\pi(B/d)^2}{4H_m^{5/2}} \left[1 + \frac{\pi}{6\bar{M}} \left(1 - \frac{1}{H_m^{3/2}} \right) \right] \quad (29)$$

and

$$\bar{F}_s^{HD} = \frac{\pi(B/d)^2 [1 + 2(1 - H_m)^2]}{2(2H_m - H_m^2)^{5/2}} \left[1 - \frac{\pi}{2\bar{M}} \frac{1 - H_m}{(2H_m - H_m^2)^{3/2}} \right] \quad (30)$$

Fig. 6 shows a comparison between the dimensionless impact forces in both HD and XPHD regimes for a given dimensionless impulse $\bar{M} = 500$ and a quite small initial compacticity, $\sigma_i = 0.05$. One can observe the impressive decrease of the maximum impact force in XPHD squeeze that sustains the interesting damping capacity of the porous layer imbibed with liquid.

Also, it is attractive to compare the maximum allowable impulses for both regimes. Hence, the maximum allowable impulse obtained in HD regime for a partial journal bearing, presented in the Appendix in Eq. (A5), is:

$$\bar{M}_{\max}^{HD} = -\frac{\pi}{6} \left(1 - \frac{1}{H_m^{3/2}} \right) \quad (31)$$

where H_{mf} is the minimum film thickness at the end of the HD squeeze process, greater than the allowable one, H_{ma} .

As a case study, the comparison can be performed using the following input data:

- B/d ratio, in both cases: $B/d = 0.5$;
- initial film/HCPL thickness, in both cases: $h_i = 1mm$;
- complex parameter of HCPL: $D = 10^{-12} \div 10^{-13} m^2$;
- initial compacticity: $\sigma_i = 0.1$;
- allowable fluid film thickness: $h_a = 5\mu m$;

Remark: the minimum final film thickness, h_f , at the end of HD squeeze process, will be considered equal with the allowable one, h_a , i.e. $H_{mf} = H_{ma}$.

The ratio between the maximum allowable impulses given by Eqs. (17) and (31), using the intervals of data, is:

$$\Re = \frac{\overline{M}_{\max}^{XPHD}}{\overline{M}_{\max}^{HD}} \approx 10 \div 10^2 \quad (32)$$

This result is in agreement with previous comparisons made in various papers [4, 12, 13].

4. CONCLUSIONS

1. A new model for squeeze under impact of a narrow circular damper with HCPL imbibed with Newtonian liquid in XPHD conditions was elaborated. The circular dampers are of particular interest because of their similarity with the commonly used squeeze film dampers.
2. Both analytic and numeric approaches were performed for partial, $\Omega = 180^\circ$, and full, $\Omega = 360^\circ$, circular squeeze damper in XPHD conditions.
3. The dimensionless impact force was analyzed for various initial compacticities and different dimensionless impulses.
4. A good correlation is obtained between the analytic and numeric approaches.
5. The results are compared with the case of classical squeeze film damper. The HD and XPHD regimes were compared in terms of dimensionless impact force and dimensionless maximum impulse. One can observe the impressive damping capacity of HCPL imbibed with liquid and an increase of over $10 \div 100$ in maximum allowable impulse for the XPHD regime versus HD regime.
6. Attractive results can lead in designing more efficient XPHD squeeze dampers.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support for the work described in this paper by National University Research Council (CNCSIS) under Grant IDEI 912.

REFERENCES

- [1] **Della Pietra, L. and Adiletta, G.** The squeeze film damper over four decades of investigations, Part I: Characteristics and operating features, *The Shock and Vibration Digest*, 2002, **34** (N 1), 3-26.
- [2] **Pascovici, M.D., Cicone, T. and Marian, V.** Squeeze process under impact, in highly compressible porous layers, imbibed with liquids, *Tribology Int*, 2009, **42**, 1433-1438.
- [3] **Pascovici, M.D., Popescu, C.S. and Marian, V.** Impact of a rigid sphere on a highly compressible porous layer imbibed with a Newtonian liquid, 2010, accepted for publication at *J. Engineering Tribology*.
- [4] **Pascovici, M.D. and Cicone, T.** Squeeze-film of unconformal, compliant and layered contacts, *Tribology Int.*, 2003, **36**, 791-799.
- [5] **Booker, J.F.** Dynamically Loaded Journal Bearings: Mobility Method Solution, *Trans. ASME, Journal of Basic Engineering*, 1965, **87**, 537-546.
- [6] **Bowden, F.P. and Tabor, D.** The friction and lubrication of solids, *Oxford: Clarendon Press*, 1950, 259-284.
- [7] **Booker, J.F.** A Table of Journal Bearing Integral, *Trans. ASME, Journal of Basic Engineering*, 1965, **87**, 533-535.
- [8] **Knox, L.D.** Squeeze film forces in a magnetic shaft suspension system, *Trans. ASME, Journal of Tribology*, 1984, **106**, 473-476.
- [9] **Pascovici, M.D.** Procedure and device for pumping by fluid dislocation. *Romanian Patent* 109469, 1994 [in Romanian].
- [10] **Han, Y., Weinbaum, S., Spaan, JAE and Vink, H.** Large – deformation analysis of the elastic recoil of fiber layers in a Brinkman medium with application to the endothelial glycocalyx, *Journal of Fluid Mechanics*, 2006, **554**, 217-235.

- [11] **Mirbod, P., Andreopoulos, Y. and Weinbaum, S.** On the generation of lift forces in random soft porous media, *Journal of Fluid Mechanics*, 2009, **619**, 147-166.
- [12] **Pascovici, M.D.** Lubrication by dislocation: a new mechanism for load carrying capacity, In: *Proceedings of second world tribology congress*, Vienna, 2001. 41 [on CD].
- [13] **Wu, Q., Andreopoulos, Y. and Weinbaum, S.** From red cells to snowboarding: a new concept for a train track, *Phys. Rev. Lett.*, 2004, **93**(19), 194501-1-4.

NOTATIONS

- B – bush length
- d – journal diameter
- d_f – fibre diameter of HCPL
- D – complex parameter of HCPL, $D = \frac{d_f^2}{16k}$
- e – eccentricity
- \bar{e} – dimensionless eccentricity, $\bar{e} = \frac{e}{h_i}$
- E – dimensionless parameter, $E = \frac{\bar{e}}{1 - \sigma_i}$
- F – imposed force
- F_s – impact force
- \bar{F}_s – dimensionless impact force, $\frac{F_s h_i^3}{\eta B d^3 V_0}$
- h – layer/film thickness
- h_0 – initial layer/film thickness
- h_i – imposed initial layer thickness

h_m	– minimal layer/film thickness
H	– dimensionless layer/film thickness, $\frac{h}{h_i}$
H_m	– minimal dimensionless layer/film thickness, $\frac{h_m}{h_i}$
H_{ma}	– minimal dimensionless allowable layer/film thickness, $\frac{h_{ma}}{h_0}$
H_{mf}	– minimal final dimensionless film thickness, $\frac{h_{mf}}{h_0}$
k	– correction constant in Kozeny-Carman law
M	– mass of impact
\overline{M}	– dimensionless impulse, $\frac{Mh_i^2V_0}{\eta B^3 d}$
\overline{M}_{\max}	– maximum allowable dimensionless impulse
Po	– permeability number, $Po = \frac{h_i^2}{D}$
\Re	– dimensionless impulses ratio
V	– impact velocity
V_0	– initial impact velocity
\overline{V}	– dimensionless impact velocity, $\frac{V}{V_0}$
ε	– HCPL porosity
η	– liquid viscosity
θ	– layer/film coordinate
σ	– HCPL compacticity
σ_0	– HCPL initial compacticity
σ_i	– HCPL imposed initial compacticity

ϕ – HCPL permeability

Ω – wrapping angle

APPENDIX

Hydrodynamic squeeze under impact for narrow journal bearings with 180° and 360° arc bush

- for 180° arc bush:

The hydrodynamic squeeze load carrying capacity of a narrow journal bearing using the Booker approximation [5] is:

$$F = \frac{\pi\eta VB^3 d}{4h_0^3 H_m^{5/2}} \quad (\text{A1})$$

Following the same procedure as for XPHD regime, using Bowden and Tabor model [6], one can find the velocity variation and the impact force:

$$V = V_0 + \frac{\pi\eta B^3 d}{6h_0^2 M} \left(1 - \frac{1}{H_m^{3/2}} \right) \quad (\text{A2})$$

and

$$F_s = \frac{\pi\eta B^3 d}{4h_0^3 H_m^{5/2}} \left[V_0 + \frac{\pi\eta B^3 d}{6h_0^2 M} \left(1 - \frac{1}{H_m^{3/2}} \right) \right] \quad (\text{A3})$$

Or, in dimensionless form, the impact force is:

$$\bar{F}_s = \frac{\pi(B/d)^2}{4H_m^{5/2}} \left[1 + \frac{\pi}{6M} \left(1 - \frac{1}{H_m^{3/2}} \right) \right] \quad (\text{A4})$$

Applying the same procedure as for XPHD regime, one can find the maximum dimensionless allowable impulse for the squeeze process held entirely in HD regime:

$$\overline{M}_{\max} = -\frac{\pi}{6} \left(1 - \frac{1}{H_m^{3/2}} \right) \quad (\text{A5})$$

where the minimum final film thickness, H_{mf} , at the end of the squeeze process must be greater than the allowable one ($H_{mf} > H_{ma}$).

- for 360° arc bush:

The hydrodynamic squeeze load carrying capacity of a narrow journal bearing corresponding with Booker HD model [5]:

$$F = \frac{\pi \eta V B^3 d}{2 h_0^3} \frac{1 + 2(1 - H_m)^2}{(2H_m - H_m^2)^{5/2}} \quad (\text{A6})$$

Further more, the velocity variation and the impact force are:

$$V = V_0 + \frac{\pi \eta B^3 d}{2 h_0^2 M} \frac{H_m - 1}{(2H_m - H_m^2)^{3/2}} \quad (\text{A7})$$

and

$$F_s = \frac{\pi \eta B^3 d [1 + 2(1 - H_m)^2]}{2 h_0^3 (2H_m - H_m^2)^{5/2}} \left[V_0 - \frac{\pi \eta B^3 d}{2 h_0^2 M} \frac{1 - H_m}{(2H_m - H_m^2)^{3/2}} \right] \quad (\text{A8})$$

Therefore, in dimensionless form, the impact force is:

$$\overline{F}_s = \frac{\pi (B/d)^2 [1 + 2(1 - H_m)^2]}{2 (2H_m - H_m^2)^{5/2}} \left[1 - \frac{\pi}{2 \overline{M}} \frac{1 - H_m}{(2H_m - H_m^2)^{3/2}} \right] \quad (\text{A9})$$

Also the dimensionless maximum impulse results:

$$\overline{M}_{\max} = \frac{\pi}{2} \frac{1 - H_m}{(2H_m - H_m^2)^{3/2}} \quad (\text{A10})$$

LIST OF FIGURES

Fig. 1 XPHD circular squeeze damper

Fig. 2 The dimensionless impact force \bar{F}_s as function of minimum dimensionless HCPL thickness H_m for different dimensionless impulses \bar{M} at a given initial compacticity $\sigma_i = 0.08$

Fig. 3 The dimensionless impact force \bar{F}_s as function of minimum dimensionless HCPL thickness H_m for different dimensionless impulses \bar{M} at a given initial compacticity $\sigma_i = 0.8$

Fig. 4 The dimensionless impact force \bar{F}_s as function of minimum dimensionless HCPL thickness H_m for a given dimensionless impulse \bar{M} and different initial compacticities

Fig. 5 Comparison between analytic and numeric results of the dimensionless impact force \bar{F}_s as function of minimum dimensionless HCPL thickness H_m for a given dimensionless impulse \bar{M} and different small initial compacticities

Fig. 6 The dimensionless impact force \bar{F}_s as function of minimum dimensionless film/HCPL thickness H_m for a given dimensionless impulse \bar{M} in both HD and XPHD regimes